OCR
RECOGNISING ACHIEVEMENT

Friday 18 January 2013 – Afternoon

A2 GCE MATHEMATICS (MEI)

4754/01 Applications of Advanced Mathematics (C4)

INSTRUCTIONS

4734010113

The examination is in two parts:

Paper A (1 hour 30 minutes)
Paper B (up to 1 hour)

Supervisors are requested to ensure that Paper B **is not issued** until Paper A has been collected in from the candidates.

Centres may, if they wish, grant a supervised break between the two parts of this examination.

Paper B should not be attached to the corresponding paper A script. For Paper A only the candidates' printed answer books, in the same order as the attendance register, should be sent for marking; the question paper should be retained in the centre or recycled. For Paper B only the question papers, on which the candidates have written their answers, should be sent for marking; the insert should be retained in the centre or recycled. Any additional sheets used must be carefully attached to the correct paper.

For Paper B (Comprehension) only.

A standard English dictionary is allowed for the comprehension.
(Dictionaries to be used in the examination must be thoroughly checked before the examination.) Full regulations are in the JCQ Regulations and Guidance booklet.

This notice must be on the Invigilator's desk at all times during the afternoon of Friday 18 January 2013.

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RECOGNISING ACHIEVEMENT

Friday 18 January 2013 – Afternoon

A2 GCE MATHEMATICS (MEI)

4754/01A Applications of Advanced Mathematics (C4) Paper A

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4754/01A
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.
- This paper will be followed by **Paper B: Comprehension**.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 Solve the equation $\frac{2x}{x+1} - \frac{1}{x-1} = 1$. **[4]**

2 Find the first four terms of the binomial expansion of $\sqrt[3]{1-2x}$. State the set of values of x for which the expansion is valid. **[6]**

3 The parametric equations of a curve are

$$x = \sin\theta, \quad y = \sin 2\theta, \quad \text{for } 0 \leqslant \theta \leqslant 2\pi.$$

(i) Find the exact value of the gradient of the curve at the point where $\theta = \frac{1}{6}\pi$. **[4]**

(ii) Show that the cartesian equation of the curve is $y^2 = 4x^2 - 4x^4$. **[3]**

4 Fig. 4 shows the curve $y = \sqrt{1 + e^{2x}}$, and the region between the curve, the x-axis, the y-axis and the line $x = 2$.

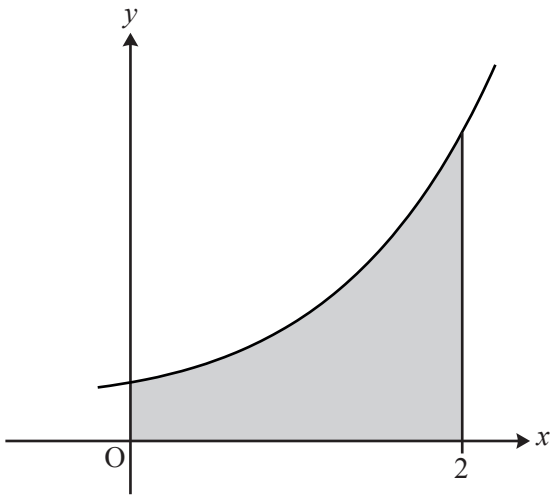

Fig. 4

(a) Find the exact volume of revolution when the shaded region is rotated through 360° about the x-axis. **[4]**

(b) **(i)** Complete the table of values, and use the trapezium rule with 4 strips to estimate the area of the shaded region. **[3]**

x	0	0.5	1	1.5	2
y		1.9283	2.8964	4.5919	

(ii) The trapezium rule for $\int_0^2 \sqrt{1 + e^{2x}}\,dx$ with 8 and 16 strips gives 6.797 and 6.823, although not necessarily in that order. Without doing the calculations, say which result is which, explaining your reasoning. **[1]**

5 Solve the equation $2\sec^2\theta = 5\tan\theta$, for $0 \leqslant \theta \leqslant \pi$. **[6]**

6 In Fig. 6, ABC, ACD and AED are right-angled triangles and BC = 1 unit. Angles CAB and CAD are θ and ϕ respectively.

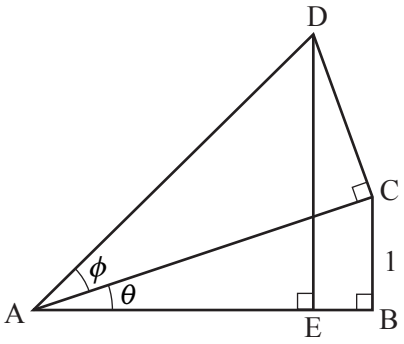

Fig. 6

(i) Find AC and AD in terms of θ and ϕ. **[2]**

(ii) Hence show that $\mathrm{DE} = 1 + \dfrac{\tan\phi}{\tan\theta}$. **[3]**

Section B (36 marks)

7 A tent has vertices ABCDEF with coordinates as shown in Fig. 7. Lengths are in metres. The O*xy* plane is horizontal.

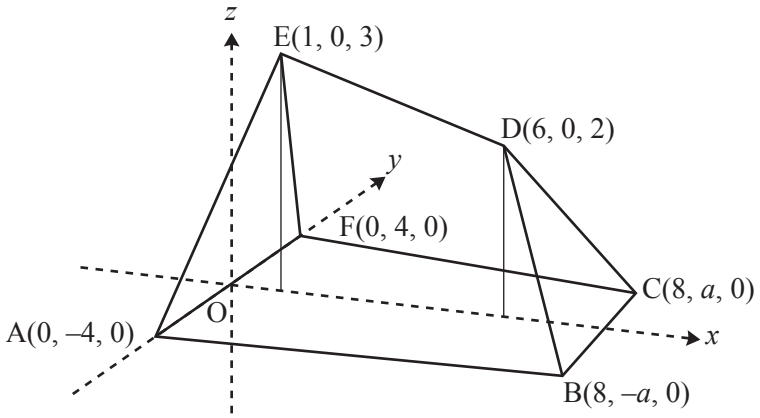

Fig. 7

(i) Find the length of the ridge of the tent DE, and the angle this makes with the horizontal. **[4]**

(ii) Show that the vector $\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$ is normal to the plane through A, D and E.

Hence find the equation of this plane. Given that B lies in this plane, find a. **[7]**

(iii) Verify that the equation of the plane BCD is $x + z = 8$.

Hence find the acute angle between the planes ABDE and BCD. **[6]**

8 The growth of a tree is modelled by the differential equation

$$10\frac{\mathrm{d}h}{\mathrm{d}t} = 20 - h,$$

where h is its height in metres and the time t is in years. It is assumed that the tree is grown from seed, so that $h = 0$ when $t = 0$.

(i) Write down the value of h for which $\frac{\mathrm{d}h}{\mathrm{d}t} = 0$, and interpret this in terms of the growth of the tree. **[1]**

(ii) Verify that $h = 20\left(1 - \mathrm{e}^{-0.1t}\right)$ satisfies this differential equation and its initial condition. **[5]**

The alternative differential equation

$$200\frac{\mathrm{d}h}{\mathrm{d}t} = 400 - h^2$$

is proposed to model the growth of the tree. As before, $h = 0$ when $t = 0$.

(iii) Using partial fractions, show by integration that the solution to the alternative differential equation is

$$h = \frac{20\left(1 - \mathrm{e}^{-0.2t}\right)}{1 + \mathrm{e}^{-0.2t}}.$$

[9]

(iv) What does this solution indicate about the long-term height of the tree? **[1]**

(v) After a year, the tree has grown to a height of 2 m. Which model fits this information better? **[3]**

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RECOGNISING ACHIEVEMENT

Friday 18 January 2013 – Afternoon

A2 GCE MATHEMATICS MEI

4754/01A Applications of Advanced Mathematics (C4) Paper A

PRINTED ANSWER BOOK

Candidates answer on this Printed Answer Book.

OCR supplied materials:
- Question Paper 4754/01A (inserted)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

Candidate forename		Candidate surname	

Centre number						Candidate number				

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- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
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INFORMATION FOR CANDIDATES

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- The paper will be followed by **Paper B: Comprehension**.

Section A (36 marks)

1	

2	

3 (i)

3 (ii)

4 (a)

4 (b) (i)

x	0	0.5	1	1.5	2
y		1.9283	2.8964	4.5919	

4 (b)(ii)

5

6 (i)

6 (ii)

Section B (36 marks)

7 (i)	

7 (ii)

7 (iii)	

8 (i)

8 (ii)

8 (iii)	
	(answer space continued on next page)

8 (iii)	**(continued)**
8 (iv)	

8 (v)	
	(answer space continued on next page)

8 (v)	(continued)

OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE

Unit **4754A:** Applications of Advanced Mathematics: Paper A

Mark Scheme for January 2013

Oxford Cambridge and RSA Examinations

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations

Annotation in scoris	Meaning
 and 	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0 M1	Method mark awarded 0, 1
A0 A1	Accuracy mark awarded 0, 1
B0 B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (eg 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

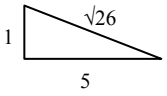
Question	Answer	Marks	Guidance
1	$\frac{2x}{x+1}-\frac{1}{x-1}=1$		
	$\Rightarrow\ 2x(x-1)-(x+1)=(x+1)(x-1)$	M1	mult throughout by $(x+1)(x-1)$ or combining fractions and mult up oe (can retain denominator throughout). Condone a single computational error provided that there is no conceptual error. Do not condone omission of brackets unless it is clear from subsequent work that they were assumed.
	$\Rightarrow\ 2x^2-3x-1=x^2-1$	A1	any fully correct multiplied out form (including say, if 1's correctly cancelled) soi
	$\Rightarrow\ x^2-3x=0=x(x-3)$	DM1	dependent on first M1.For any method leading to both solutions. Collecting like terms and forming quadratic (= 0) and attempting to solve *(provided that it is a quadratic and $b^2-4ac\geq 0$).Using either correct quadratic equation formula (can be error when substituting), factorising (giving correct x^2 and constant terms when factors multiplied out), completing the square oe soi.*
	$\Rightarrow\ x=0$ or 3	A1	for both solutions www.
			SC B1 for $x=0$ (or $x=3$) without any working **SC** B2 for $x=0$ (or $x=3$) without above algebra but showing that they satisfy equation **SC** M1A1M0 SCB1 for first two stages followed by stating $x=0$ **SC** M1A1M0 SCB1 for first two stages and cancelling x to obtain $x=3$ only.
		[4]	

Question	Answer	Marks	Guidance
2	$\sqrt[3]{1-2x} = (1-2x)^{1/3}$	B1	$n = 1/3$ only. Do not MR for $n \neq 1/3$
	$= 1 + \frac{1}{3}(-2x) + \frac{\frac{1}{3}(-\frac{2}{3})}{2!}(-2x)^2 + \frac{\frac{1}{3}(-\frac{2}{3})(-\frac{5}{3})}{3!}(-2x)^3 + \ldots$	M1	all four **correct unsimplified** binomial coeffs (not nCr) soi condone absence of brackets only if it is clear from subsequent work that they were assumed
		B1	$1 - \frac{2}{3}x$ www in this term
	$= 1 - \frac{2}{3}x - \frac{4}{9}x^2 - \frac{40}{81}x^3 + \ldots$	B1	…. $-\frac{4}{9}x^2$ www in this term (not if used $2x$ for $(-2x)$ throughout)
		B1	….. $-\frac{40}{81}x^3$ www in this term If there is an error in say the third coeff of the expansion then M0 B1B0B1 is possible.
	Valid for $-½ < x < ½$ or $\lvert x\rvert < ½$	B1	Independent of expansion Allow ≤'s (valid in this case) or a combination. Condone also, say, $-½ < \lvert x\rvert < ½$ but not $x < ½$ or $-1 < 2x < 1$ or $-½ > x > ½$
		[6]	

Question	Answer	Marks	Guidance
3 (i)	$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{2\cos 2\theta}{\cos\theta}$	M1 A1	their $dy/d\theta$ / their $dx/d\theta$ www correct (can isw)
	When $\theta = \pi/6$ $= \frac{dy}{dx} = \frac{2\cos(\pi/3)}{\cos(\pi/6)}$ $= \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}}$	DM1 A1	subst $\theta = \pi/6$ in theirs oe exact only, www (but not $1/\sqrt{3}/2$)
	……		……..
	OR		
	$y = 2x\sqrt{(1-x^2)}$ $\frac{dy}{dx} = -2x^2(1-x^2)^{-1/2} + 2(1-x^2)^{1/2}$	M1 A1	full method for differentiation including product rule and function of a function oe oe cao (condone lack of consideration of sign)
	$at\theta = \pi/6, \sin\pi/6 = 1/2$	DM1	subst $\sin\pi/6 = ½$ in theirs
	$\frac{dy}{dx} = \frac{-2}{4}(1-\frac{1}{4})^{-1/2} + 2(\frac{3}{4})^{1/2} = \frac{2}{\sqrt{3}}$	A1	oe ,exact only, www (but not $1/\sqrt{3}/2$)
		[4]	
3 (ii)	$y = \sin 2\theta = 2 \sin\theta \cos\theta$	M1	using $\sin 2\theta = 2 \sin\theta \cos\theta$
	$\Rightarrow \quad y^2 = 4 \sin^2\theta \cos^2\theta = 4x^2(1-x^2)$ $= 4x^2 - 4x^4$ *	M1 A1	using $\cos^2\theta = 1 - \sin^2\theta$ to eliminate $\cos\theta$ **AG** need to see sufficient working or A0.
		[3]	

Question			Answer	Marks	Guidance
4	(a)		$V = \int_0^2 \pi y^2 \mathrm{d}x = \int_0^2 \pi(1 + \mathrm{e}^{2x})\mathrm{d}x$	M1	$\int_0^2 \pi(1 + \mathrm{e}^{2x})\mathrm{d}x$ limits must appear but may be later condone omission of *dx* if intention clear
			$= \pi\left[x + \frac{1}{2}\mathrm{e}^{2x}\right]_0^2$	B1	$\left[x + \frac{1}{2}\mathrm{e}^{2x}\right]$ independent of π and limits
			$= \pi(2 + ½\,\mathrm{e}^4 - ½)$	DM1	dependent on first M1.Need **both** limits substituted in their integral of the form $ax + b\,e^{2x}$, where a, b non-zero constants. Accept answers including e^0 for M1. Condone absence of π for M1 at this stage
			$= ½\,\pi(3 + \mathrm{e}^4)$	A1 **[4]**	cao exact only
4	(b)	(i)	$x = 0$, $y = 1.4142$; $x = 2$, $y = 7.4564$	B1	1.414, 7.456 or better
			$A = 0.5/2\{(1.4142 + 7.4564) + 2(1.9283 + 2.8964 + 4.5919)\}$	M1	correct formula seen (can be implied by correct intermediate step eg 27.7038../4)
			$= 6.926$	A1 **[3]**	6.926 or 6.93 (do not allow more dp)
4	(b)	(ii)	8 strips: 6.823, 16 strips: 6.797		
			Trapezium rule overestimates this area, but the overestimate gets less as the no of strips increases.	B1 **[1]**	oe

Question	Answer	Marks	Guidance
5	$2\sec^2\theta = 5\tan\theta$		
	$\Rightarrow \quad 2(1+\tan^2\theta) = 5\tan\theta$	M1	$\sec^2\theta = 1+\tan^2\theta$ **used**
	$\Rightarrow \quad 2\tan^2\theta - 5\tan\theta + 2 = 0$	A1	correct quadratic **oe**
	$\Rightarrow \quad (2\tan\theta - 1)(\tan\theta - 2) = 0$	M1	solving their quadratic for $\tan\theta$ (follow rules for solving as in Question 1 [*,*]
	$\Rightarrow \quad \tan\theta = ½$ or 2	A1	www
	$\Rightarrow \quad \theta = 0.464,$	A1	first correct solution (or better)
	1.107	A1	second correct solution (or better) and no others in the range Ignore solutions outside the range. **SC** A1 for both 0.46 and 1.11 **SC** A1 for both 26.6° and 63.4° (or better) Do not award **SC**s if there are extra solutions in range.
	……		……
	OR		
	$2/\cos^2\theta = 5\sin\theta/\cos\theta$	M1	**using** both sec = 1/cos and tan = sin/cos
	$\Rightarrow 2\cos\theta = 5\sin\theta\cos^2\theta, \cos\theta \neq 0$	A1	correct one line equation $2 - 5\sin\theta\cos\theta = 0$ or $2\cos\theta = 5\sin\theta\cos^2\theta$ oe (or common denominator). Do not need $\cos\theta \neq 0$ at this stage.
	$\Rightarrow \cos\theta(2 - 5\sin\theta\cos\theta) = 0$		
	$\Rightarrow \cos\theta = 0, or \sin 2\theta = 0.8$	M1	**using** $\sin 2\theta = 2\sin\theta\cos\theta$ oe eg $2 = 5\sin\theta\sqrt{(1-\sin^2\theta)}$ and squaring
	$\Rightarrow \sin 2\theta = 0.8$	A1	$\sin 2\theta = 0.8$ or, say, $25\sin^4\theta - 25\sin^2\theta + 4 = 0$
	$\Rightarrow 2\theta = 0.9273 or 2.2143$		
	$\Rightarrow \theta = 0.464,$	A1	first correct solution (or better)
	1.107	A1	second correct solution (or better) and no others in range Ignore solutions outside the range SCs as above
		[6]	

Question	Answer	Marks	Guidance
6 (i)	AC = cosec θ $\Rightarrow$ AD = cosecθ sec φ	M1 A1 **[2]**	or 1/sin θ oe but not if a fraction within a fraction
6 (ii)	DE = AD sin ($\theta + \varphi$) = cosec θ sec φ sin ($\theta + \varphi$) = cosecθ sec φ(sin θ cos φ + cos θ sin φ) $= \frac{\sin\theta\cos\varphi + \cos\theta\sin\varphi}{\sin\theta\cos\varphi}$ $= 1 + \frac{\cos\theta}{\sin\theta}\frac{\sin\varphi}{\cos\varphi}$ = 1 + tan φ / tan θ * …….. **OR** equivalent, eg from DE = CB + CD cos θ = 1 + CD cos θ = 1 + AD sin φ cos θ = 1 + cosec θ sec φ sin φ cos θ = 1 + tanφ /tanθ*	M1 M1 A1 M1 M1 A1 **[3]**	AD sin($\theta + \varphi$) with substitution for their AD correct compound angle formula used **Do not** award both M marks unless they are part of the same method. (They may appear in either order.) simplifying using tan = sin/cos. A0 if no intermediate step as **AG** …….. from triangle formed by using X on DE where CX is parallel to BE to get DX = CD cos θ and CB = 1 (oe trigonometry) substituting for both CD = AD sinφ and their AD oe to reach an expression for DE **in terms of θ and φ only** (M marks must be part of same method) **AG** simplifying to required form
7 (i)	DE = $\sqrt{[(-5)^2 + 0^2 + 1^2]} = \sqrt{26}$  cos θ = 5/√26 oe $\Rightarrow$ θ = 11.3°	M1 A1 M1 A1 **[4]**	oe oe using scalar products eg –5**i** + **k** with **i** oe or better (or 168.7°). Allow radians.

Question	Answer	Marks	Guidance
7 (ii)	$\overrightarrow{AE} = \begin{pmatrix}1\\4\\3\end{pmatrix}$, $\overrightarrow{ED} = \begin{pmatrix}5\\0\\-1\end{pmatrix}$	B1	**two** relevant direction vectors (or $6\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$ oe)
	$\begin{pmatrix}1\\4\\3\end{pmatrix}.\begin{pmatrix}1\\-4\\5\end{pmatrix} = 1-16+15 = 0$	B1	scalar product with a **direction** vector in the plane (including evaluation and = 0) (**OR** M1 forms vector cross product with at least two correct terms in solution)
	$\begin{pmatrix}5\\0\\-1\end{pmatrix}.\begin{pmatrix}1\\-4\\5\end{pmatrix} = 5+0-5 = 0$	B1	scalar product with second **direction** vector, with evaluation. (following **OR** above, A1 all correct ie a multiple of $\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$) (NB finding only one direction vector and its scalar product is B1 only.)
	$\Rightarrow$ $\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$ is normal to AED		
	$\Rightarrow$ eqn of AED is $\begin{pmatrix}x\\y\\z\end{pmatrix}.\begin{pmatrix}1\\-4\\5\end{pmatrix} = \begin{pmatrix}0\\-4\\0\end{pmatrix}.\begin{pmatrix}1\\-4\\5\end{pmatrix}$	M1	for $x - 4y + 5z = c$ oe
	$\Rightarrow$ $x - 4y + 5z = 16$	A1	M1A0 for $\mathbf{i} - 4\mathbf{j} + 5\mathbf{k} = 16$
	B lies in plane if $8 - 4(-a) + 5.0 = 16$	M1	allow M1 for subst in their plane or if found from say scalar product of normal with vector EB can also get M1A1
	$\Rightarrow$ $a = 2$	A1	**For first five marks above** **SC1**, if states, 'if $\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$ is normal then of form $x - 4y + 5z = c$' and substitutes one coordinate gets M1A1, then substitutes other two coordinates A2 (not A1,A1).Then states so $\begin{pmatrix}1\\-4\\5\end{pmatrix}$ is normal can get B1 provided that there is a clear argument ie M1A1A2B1. Without a clear argument this is B0. **SC2,** if finds two relevant vectors, B1 and then finds equation of the plane from vector form, $r = a + \mu b + \lambda c$ gets B1. Eliminating parameters B1cao. If then states 'so $\begin{pmatrix}1\\-4\\5\end{pmatrix}$ is normal' can get B1 (4/5).
		[7]	

Question			Answer	Marks	Guidance
7	(iii)		D: 6 + 2 = 8 B: 8 + 0 = 8 C: 8 + 0 = 8 ⇒ plane BCD is $x + z = 8$ Angle between $\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$ and $\mathbf{i} + \mathbf{k}$ is θ ⇒ $\cos\theta = (1\times1+(-4)\times0 + 5\times 1)/\sqrt{42}\sqrt{2} = 6/\sqrt{84}$ ⇒ $\theta = 49.1°$	 B2,1,0 M1 M1 A1 A1 **[6]**	 or any valid method for finding $x + z = 8$ gets M1A1 between two correct relevant vectors complete method (including cosine) (**for M1** ft their normal(s) to their plane(s)) allow correct substitution or $\pm6/\sqrt{84}$, correct only or 0.857 radians (or better) acute only
8	(i)		h = 20, stops growing	B1 **[1]**	**AG** need interpretation
	(ii)		$h = 20 - 20e^{-t/10}$ $dh/dt = 2e^{-t/10}$ $20\,e^{-t/10} = 20 - 20(1 - e^{-t/10}) = 20 - h$ $=10dh/dt$ when $t = 0$, $h = 20(1 - 1) = 0$ …… **OR** verifying by integration $\int\frac{dh}{20-h}=\int\frac{dt}{10}$ $\Rightarrow -\ln(20-h)=0.1t+c$ $h=0,t=0,\Rightarrow c=-\ln 20$ $\Rightarrow \ln(20-h)=-0.1t+\ln 20$ $\Rightarrow \ln(\frac{20-h}{20})=-0.1t$ $\Rightarrow 20-h=20e^{-0.1t}$ $\Rightarrow h=20(1-e^{-0.1t})$	M1A1 M1 A1 B1 M1 A1 B1 M1 A1 **[5]**	differentiation (for M1 need $ke^{-t/10}$, k const) oe eg $20 - h = 20 - 20(1 - e^{-t/10}) = 20e^{-t/10}$ $= 10dh/dt$ (showing sides equivalent) initial conditions …….. sep correctly and intending to integrate correct result (condone omission of c, although no further marks are possible) condone ln (h – 20) as part of the solution at this stage constant found from expression of correct form (at any stage) but B0 if say $c = \ln(-20)$ (found using $\ln(h - 20)$) combining logs and anti-logging (correct rules) correct form (do not award if B0 above)

Question	Answer	Marks	Guidance
8 (iii)	$\frac{200}{(20+h)(20-h)} = \frac{A}{20+h} + \frac{B}{20-h}$		
	$\Rightarrow \quad 200 = A(20-h) + B(20+h)$	M1	cover up, substitution or equating coeffs
	$h = 20 \Rightarrow 200 = 40B$, $B = 5$	A1	
	$h = -20 \Rightarrow 200 = 40A$, $A = 5$	A1	
	$200\ \mathrm{d}h/\mathrm{d}t = 400 - h^2$		
	$\Rightarrow \quad \int \frac{200}{400-h^2}\,dh = \int dt$	M1	separating variables and intending to integrate (condone sign error)
	$\Rightarrow \quad \int (\frac{5}{20+h} + \frac{5}{20-h})dh = \int dt$		substituting partial fractions
	$\Rightarrow \quad 5\ln(20+h) - 5\ln(20-h) = t + c$	A1	ft their A,B, condone absence of c, Do not allow ln(h-20) for A1.
	When $t = 0$, $h = 0 \Rightarrow 0 = 0 + c \Rightarrow c = 0$	B1	cao need to **show** this. c can be found at any stage. **NB c = ln(–1)** (from ln(h –20)) or similar **scores B0.**
	$\Rightarrow \quad 5\ln\frac{20+h}{20-h} = t$		
	$\Rightarrow \quad \frac{20+h}{20-h} = e^{t/5}$	M1	anti-logging an equation of the correct form . Allow if c = 0 clearly stated (provided that c = 0) even if B mark is not awarded, but do not allow if c omitted. Can ft their c.
	$\Rightarrow \quad 20 + h = (20-h)e^{t/5} = 20\,e^{t/5} - h\,e^{t/5}$		
	$\Rightarrow \quad h + h\,e^{t/5} = 20\,e^{t/5} - 20$		
	$\Rightarrow \quad h(e^{t/5} + 1) = 20(e^{t/5} - 1)$	DM1	making h the subject, dependent on previous mark
	$\Rightarrow \quad h = \frac{20(e^{t/5}-1)}{e^{t/5}+1}$		**NB** method marks can be in either order, in which case the dependence is the other way around.(In which case, 20 + h is divided by 20 – h first to isolate h).
	$\Rightarrow \quad h = \frac{20(1-e^{-t/5})}{1+e^{-t/5}}$ *	A1	**AG must have obtained B1** (for c) in order to obtain final A1**.**
		[9]	
8 (iv)	As $t \to \infty$, $h \to 20$. So long-term height is 20m.	B1	www
		[1]	
8 (v)	1st model $h = 20(1 - e^{-0.1}) = 1.90..$	B1	Or 1st model h = 2 gives t = 1.05..
	2nd model $h = 20(e^{1/5} - 1)/(e^{1/5} + 1) = 1.99..$	B1	2nd model h = 2 gives t = 1.003..
	so 2nd model fits data better	B1 dep	dep previous B1s correct
		[3]	

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Friday 18 January 2013 – Afternoon

A2 GCE MATHEMATICS (MEI)

4754/01B Applications of Advanced Mathematics (C4) Paper B: Comprehension

QUESTION PAPER

Candidates answer on the Question Paper.

OCR supplied materials:
- Insert (inserted)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator
- Rough paper

Duration: Up to 1 hour

* 4 7 5 4 0 1 B *

* 4 7 3 4 0 4 0 1 1 3 *

Candidate forename		Candidate surname	

Centre number						Candidate number				

INSTRUCTIONS TO CANDIDATES

- The insert will be found in the centre of this document.
- Write your name, centre number and candidate number in the boxes above. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Write your answer to each question in the space provided. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- The insert contains the text for use with the questions.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

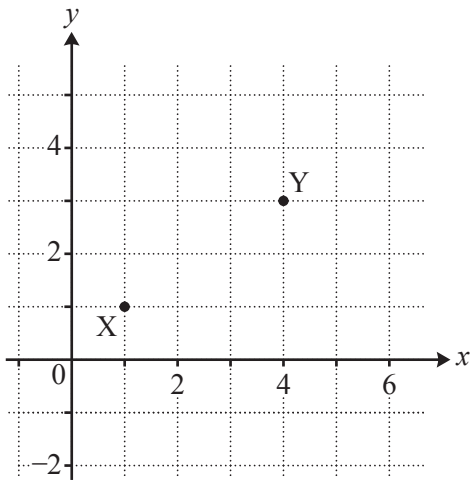
- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You may find it helpful to make notes and do some calculations as you read the passage.
- You are **not** required to hand in these notes with your question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **18**.
- This document consists of **8** pages. Any blank pages are indicated.

BLANK PAGE

PLEASE DO NOT WRITE ON THIS PAGE

1 On the grid below mark all three possible positions of the point P with integer coordinates for which $t(P,X) = 4$ and $t(P,Y) = 3$. **[3]**

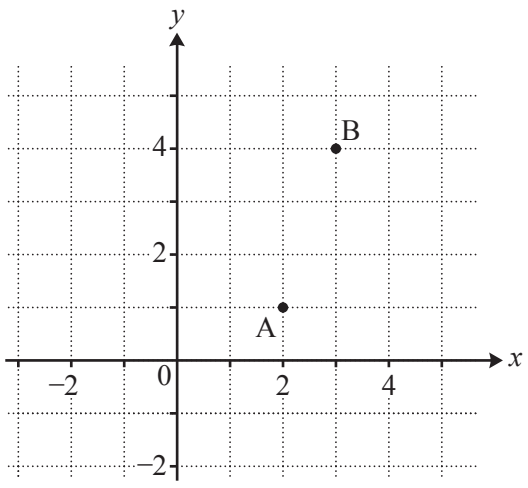
1



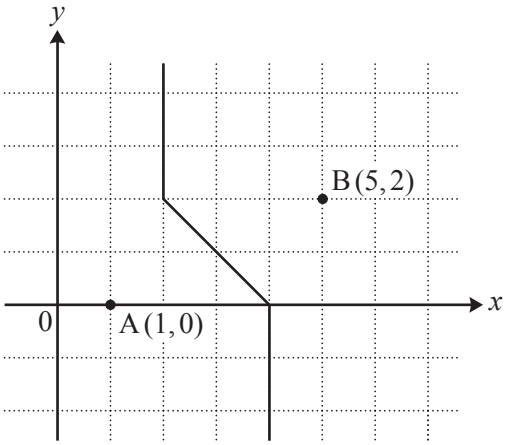
2 This question is concerned with generalised taxicab geometry.

On the grid below, show the locus of a point P where $t(P,A) = t(P,B)$. **[3]**

2

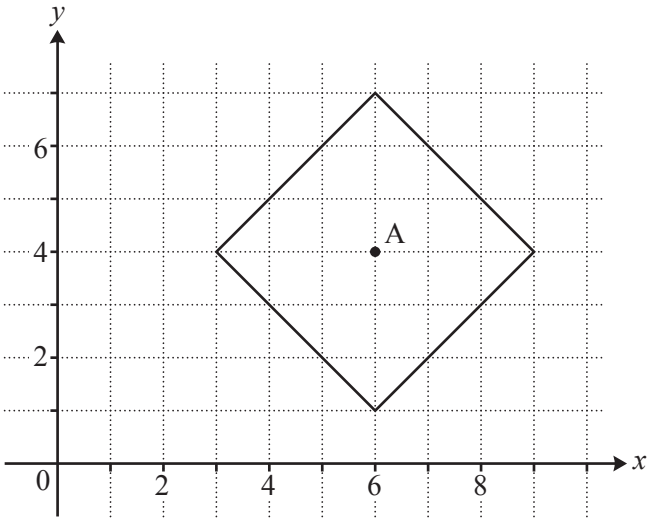


3 **(i)** Describe the following locus of a point P, using the notation $t(P, A)$ and $t(P, B)$ as appropriate.



[1]

(ii) Describe the following locus of a point P, using the notation $t(P, A)$ as appropriate.



[1]

3 (i)	
3 (ii)	

PLEASE DO NOT WRITE IN THIS SPACE

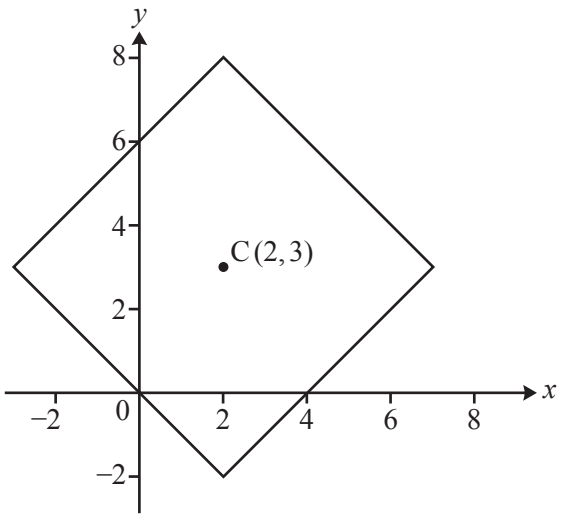
4 Referring to Fig. 5, or otherwise, find the value of $n(4,4)$. **[2]**

4	

5 In lines 54 and 55 it says there are 35 minimum distance routes from A $(0,0)$ to B $(4,3)$. Determine how many of these routes pass through the point with coordinates $(3,2)$, explaining your reasoning. **[2]**

5	

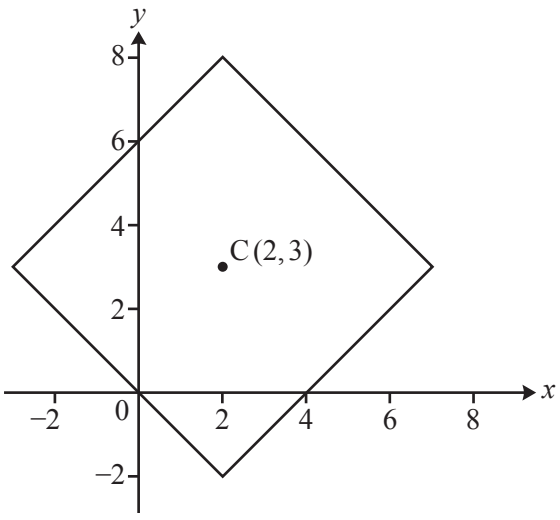
6 Fig. 7 is reproduced below.



(i) Two points on this locus have x-coordinate -0.7. Write down the coordinates of each of these points. **[2]**

(ii) In lines 77 to 78 it says "adding a second taxicab circle with centre $(2,0)$ and radius 2 shows that in generalised taxicab geometry two different circles can have an infinite number of points in common!"

On the copy of Fig. 7 given below, draw the taxicab circle with centre $(2,0)$ and radius 2. **[1]**

6 (i)	
6 (ii)	

7 In lines 23 and 24 it says that "if the Pythagorean distance between two points A and B is $\mathrm{d}(\mathrm{A},\mathrm{B})$ then the taxicab distance satisfies the inequalities $\mathrm{d}(\mathrm{A},\mathrm{B}) \leqslant \mathrm{t}(\mathrm{A},\mathrm{B}) \leqslant \sqrt{2} \times \mathrm{d}(\mathrm{A},\mathrm{B})$."

This question is about using this result in generalised taxicab geometry.

(i) Given that A is the point $(0,0)$, describe all possible positions of B for which $\mathrm{d}(\mathrm{A},\mathrm{B}) = \mathrm{t}(\mathrm{A},\mathrm{B})$. **[1]**

(ii) Given that A is the point $(0,0)$, describe all possible positions of B for which $\mathrm{t}(\mathrm{A},\mathrm{B}) = \sqrt{2} \times \mathrm{d}(\mathrm{A},\mathrm{B})$. **[2]**

7 (i)	
7 (ii)	

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Friday 18 January 2013 – Afternoon

A2 GCE MATHEMATICS (MEI)

4754/01B Applications of Advanced Mathematics (C4) Paper B: Comprehension

INSERT

Duration: Up to 1 hour

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Taxicab geometry

Introduction

Fig. 1 shows part of the road map of an imaginary town called Newtown.

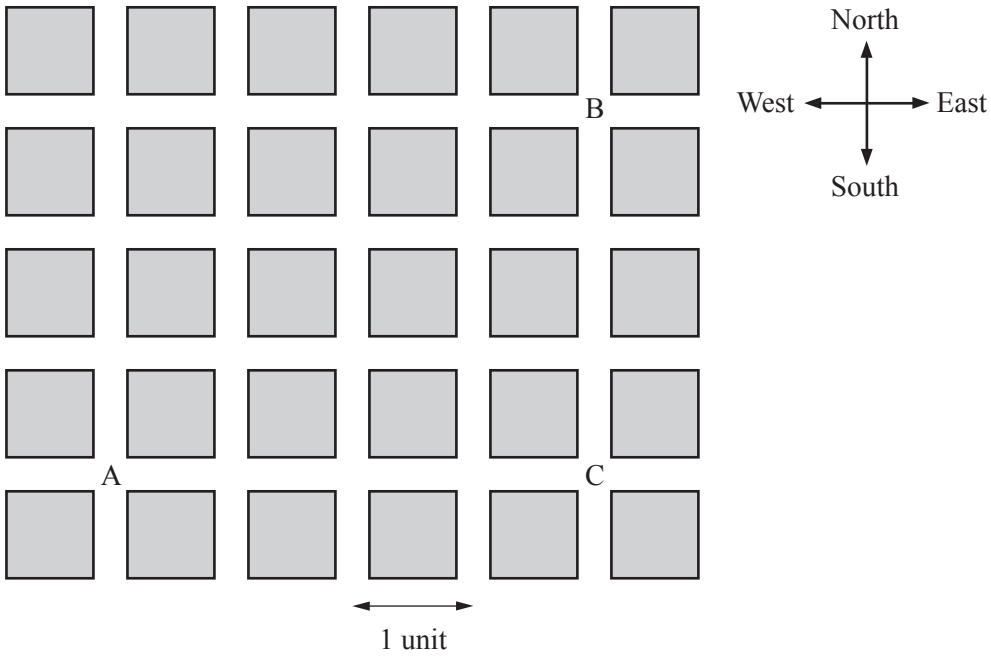

Fig. 1

Newtown's buildings are grouped in equal-sized square blocks. The roads between the blocks run in north-south and east-west directions and traffic can travel along every road in both directions.

Imagine you want to take a taxi from point A to point B. If the taxi travelled east from A to C and then north from C to B, the total distance travelled would be 7 units. Many other routes from A to B are also 7 units in length but no route is shorter. This shortest distance is called the *taxicab distance* from A to B and the related mathematics is called *taxicab geometry*.

This article introduces some of the mathematics of taxicab geometry.

Introducing the notation

Fig. 2 shows part of the road map of Newtown and one particular bus route with bus stops at positions M and N. Imagine you are at position L and you wish to catch a bus at one of these bus stops. Which is closer?

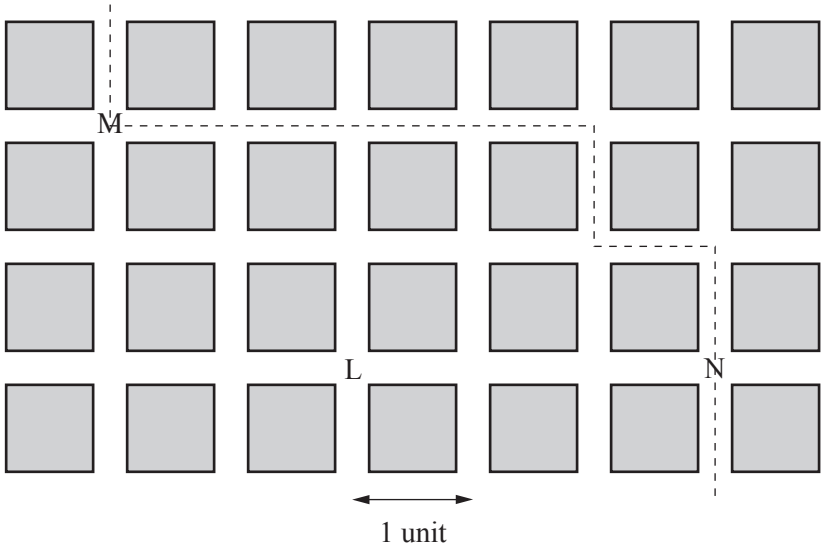

Fig. 2

By Pythagoras's Theorem, the straight line distance, measured in units as shown in Fig. 2, from L to M is $2\sqrt{2}$. This is expressed using the notation $\mathrm{d}(\mathrm{L},\mathrm{M}) = 2\sqrt{2}$. Similarly, $\mathrm{d}(\mathrm{L},\mathrm{N}) = 3$. In terms of straight line distances, M is closer than N since $\mathrm{d}(\mathrm{L},\mathrm{M}) < \mathrm{d}(\mathrm{L},\mathrm{N})$.

For a pedestrian, who is constrained to walking along roads, it is the taxicab distance rather than the Pythagorean distance that is important. The taxicab distance from L to M is 4. This is expressed using the notation $\mathrm{t}(\mathrm{L},\mathrm{M}) = 4$. Similarly $\mathrm{t}(\mathrm{L},\mathrm{N}) = 3$. For a pedestrian at L, since $\mathrm{t}(\mathrm{L},\mathrm{N}) < \mathrm{t}(\mathrm{L},\mathrm{M})$, N is closer than M.

This is an example of a situation in which $\mathrm{d}(\mathrm{L},\mathrm{M}) < \mathrm{d}(\mathrm{L},\mathrm{N})$ but $\mathrm{t}(\mathrm{L},\mathrm{M}) > \mathrm{t}(\mathrm{L},\mathrm{N})$.

In general, if the Pythagorean distance between two points A and B is $\mathrm{d}(\mathrm{A},\mathrm{B})$ then the taxicab distance satisfies the inequalities $\mathrm{d}(\mathrm{A},\mathrm{B}) \leqslant \mathrm{t}(\mathrm{A},\mathrm{B}) \leqslant \sqrt{2} \times \mathrm{d}(\mathrm{A},\mathrm{B})$.

Minimum distance routes

In Fig. 1 the Pythagorean distance between the points A and B is 5. There is only one straight line segment from A to B; its length is 5.

However, this uniqueness property does not hold when considering the taxicab distance. In Fig. 1, the taxicab distance from A to B is 7. There are several routes from A to B which have this minimum distance; these are called *minimum distance routes*.

How many minimum distance routes are there from A to B?

In order to answer this question, the road grid is replaced by a coordinate system as shown in Fig. 3. The x-axis represents the west-east direction and the y-axis represents the south-north direction. Point A has coordinates $(0,0)$ and point B has coordinates $(4,3)$. The roads are shown by the grid lines.

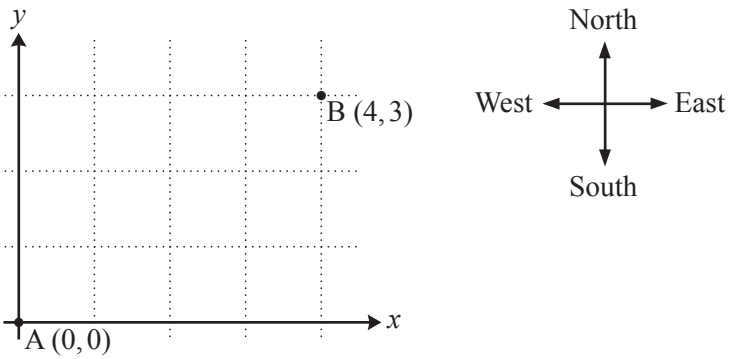

Fig. 3

Clearly, no minimum distance route from A to any point in the first quadrant will involve any motion in a westerly or southerly direction.

There is only one minimum distance route from A to any point on the x-axis or to any point on the y-axis.

There are two ways of reaching the point with coordinates $(1,1)$ along minimum distance routes as follows.

$$(0,0) \rightarrow (1,0) \rightarrow (1,1)$$

$$(0,0) \rightarrow (0,1) \rightarrow (1,1)$$

The numbers of minimum distance routes from A to the points mentioned above are shown in Fig. 4.

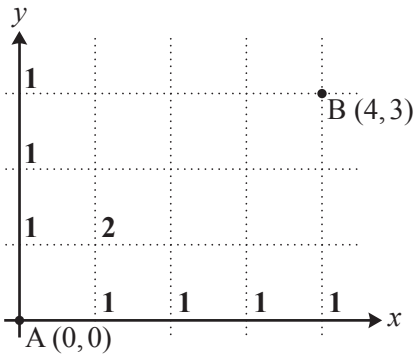

Fig. 4

The final step on a minimum distance route from A to the point $(2,1)$ must be either from $(2,0)$ to $(2,1)$ or from $(1,1)$ to $(2,1)$. There is 1 minimum distance route from A to $(2,0)$ and there are 2 minimum distance routes from A to $(1,1)$. Each of these routes can be continued to $(2,1)$ in only one way. Since all of these routes are different, the number of minimum distance routes from A to $(2,1)$ is 3.

This reasoning can be extended to other grid points. The notation $\mathrm{n}(p,q)$ is used to denote the number of minimum distance routes from $(0,0)$ to (p,q), where p and q are non-negative integers. The following rules apply for $p \geqslant 1, q \geqslant 1$.

$$\mathrm{n}(p,0) = 1$$
$$\mathrm{n}(0,q) = 1$$
$$\mathrm{n}(p,q) = \mathrm{n}(p-1,q) + \mathrm{n}(p,q-1)$$

These rules give the numbers of minimum distance routes shown in Fig. 5.

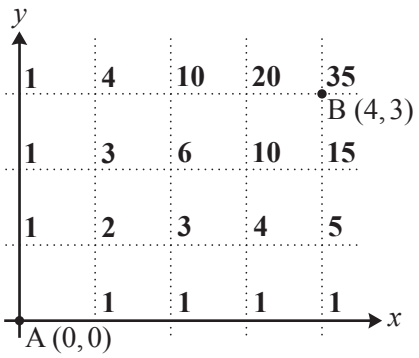

Fig. 5

So the answer to the question of how many minimum distance routes there are from $\mathrm{A}(0,0)$ to $\mathrm{B}(4,3)$ is 35.

Generalised taxicab geometry

The mathematical model of taxicab geometry described so far has been motivated by a system of roads and junctions. In this system there is a finite number of uniformly spaced parallel and perpendicular roads and all journeys start and end at junctions.

The mathematical ideas can be generalised by defining the taxicab distance for any two points in the *x*-*y* plane. In this generalised version, the points are not necessarily grid points.

Fig. 6 shows two points, $\mathrm{R}(x_1, y_1)$ and $\mathrm{S}(x_2, y_2)$, in the *x*-*y* plane.

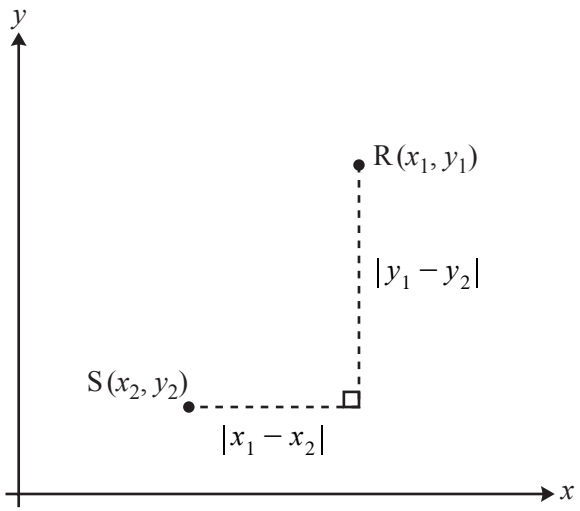

Fig. 6

The taxicab distance, $\mathrm{t}(\mathrm{R}, \mathrm{S})$, is defined as $|x_1 - x_2| + |y_1 - y_2|$. Thus the taxicab distance is still defined as the sum of the distances between the points in the *x*- and *y*-directions.

For example, the taxicab distance between the points with coordinates $(2.1, 1)$ and $(3.9, 4.3)$ is

$$\begin{aligned} & |2.1 - 3.9| + |1 - 4.3| \\ = \; & |-1.8| + |-3.3| \\ = \; & 1.8 + 3.3 \\ = \; & 5.1. \end{aligned}$$

Similarly the taxicab distance between $(-1.1, 1.4)$ and $(3.2, -0.8)$ is

$$|-1.1 - 3.2| + |1.4 - (-0.8)| = |-4.3| + |2.2| = 4.3 + 2.2 = 6.5.$$

This definition of distance produces some surprising geometric results, as will be seen below.

Fig. 7 shows a fixed point $C(2,3)$ and the locus of the point P satisfying $t(P,C) = 5$. The coordinates of every point $P(x,y)$ on this locus satisfy the equation $|x-2|+|y-3| = 5$.

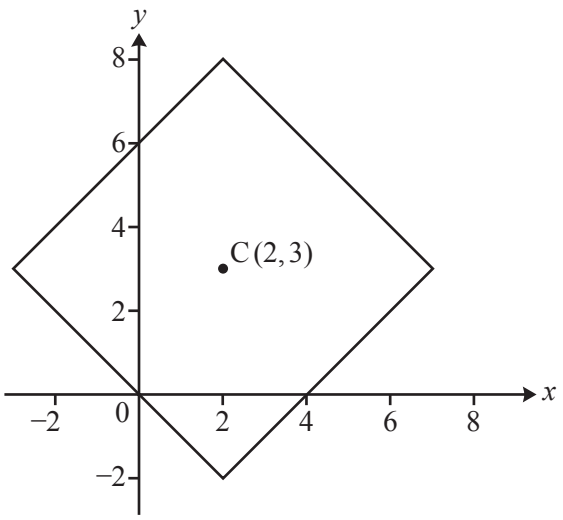

Fig. 7

Since all points are at a fixed taxicab distance from C, this is a taxicab 'circle' in this geometry! The circle has a taxicab 'radius' of 5.

Furthermore, adding a second taxicab circle with centre $(2,0)$ and radius 2 shows that in generalised taxicab geometry two different circles can have an infinite number of points in common!

Now consider the locus of a point Q which is 'equidistant' from two fixed points $A(0,0)$ and $B(8,6)$.

Fig. 8.1 shows the set of points Q satisfying $d(Q,A) = d(Q,B)$; this is the familiar perpendicular bisector of the line segment AB.

Fig. 8.2 shows the set of points Q satisfying $t(Q,A) = t(Q,B)$; so in generalised taxicab geometry the locus is quite different.

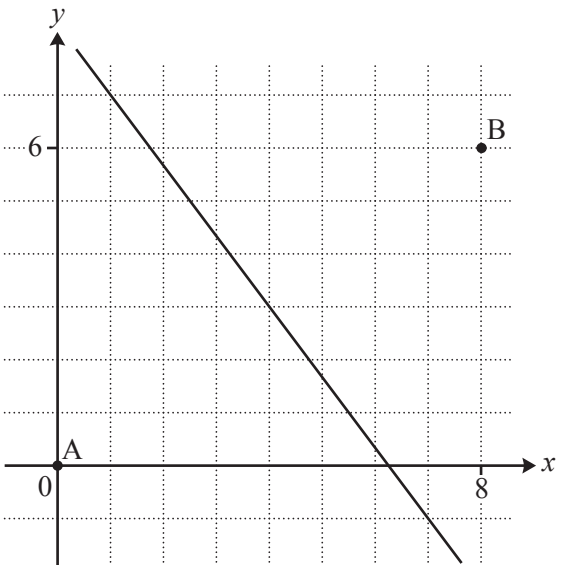

Fig. 8.1

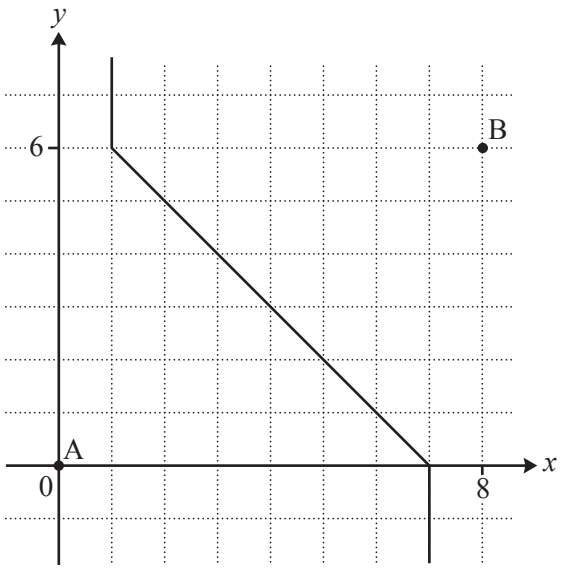

Fig. 8.2

Conclusion

In the natural world it is often appropriate to apply Pythagoras's Theorem to calculate the distance between two points. However, in urban geography, where there are obstacles such as buildings to be considered, taxicab geometry is often a more useful mathematical model.

In this article several simplifying assumptions have been made. For example, the imaginary town is laid out in a square grid, all roads are traversable in both directions and that the rate of progress along every route is uniform. Although these clearly do not exactly match any real cities, Fig. 9 below, a map of Manhattan in New York, suggests that, for some cities, some form of taxicab geometry can provide a good mathematical model.

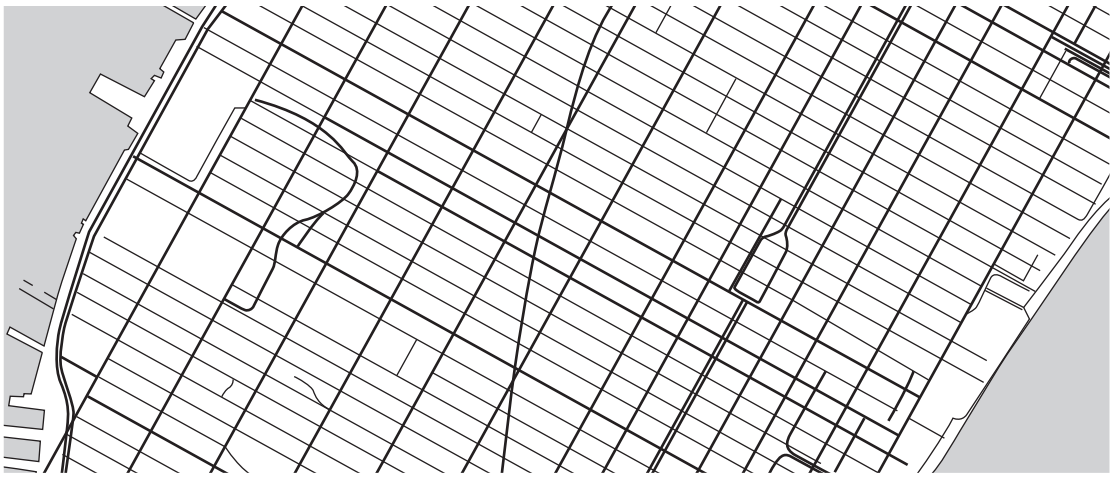

Fig. 9

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Mark Scheme for January 2013

Oxford Cambridge and RSA Examinations

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations

Annotation in scoris	Meaning
 and	
	Benefit of doubt
	Follow through
	Ignore subsequent working
	Method mark awarded 0, 1
	Accuracy mark awarded 0, 1
	Independent mark awarded 0, 1
	Special case
	Omission sign
	Misread
	Highlighting

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (eg 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

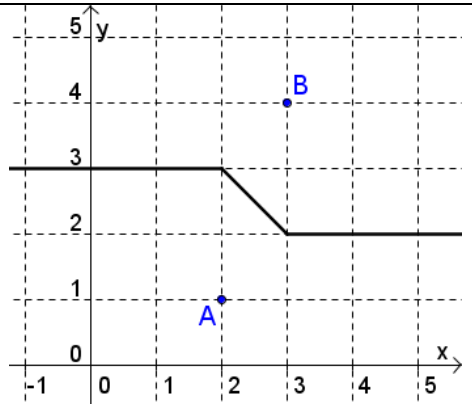
If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

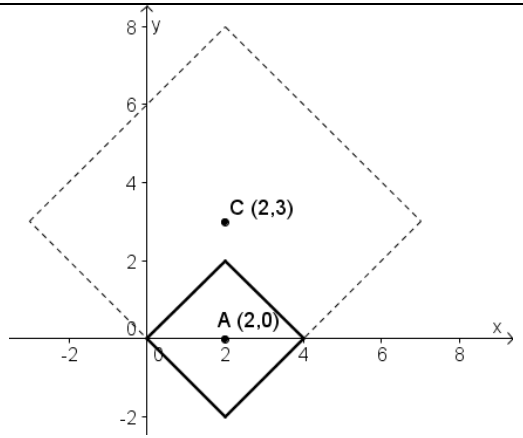
If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1			(4,0) and (5,1) and (2,4)	B3 **[3]**	All three points correct (and no additional points) Accept a list of coordinates.	SC B1 any correct point SC B2 any two correct points (and no more than two incorrect additional points) B0 if points unclear
2				B3 **[3]**	B3: Exactly as shown (with line extended beyond (4,2) and before (0,3)) Award B1 for a locus including one partially correct line segment or at least two correct discrete points in that segment Award B2 for a locus that contains parts of all three correct line segments (or at least two discrete points within each of them) and no additional incorrect points.	
3	(i)		$t(P,A) = t(P,B)$	B1 **[1]**	(not t(P,A)=t(P,B)=3)	Accept words, such as, [distance] t(P,A) is equal to [distance] t(P,B).
3	(ii)		$t(P,A) = 3$	B1 **[1]**		Accept words such as, all the points with [distance] t(P,A) equal to 3.
4			Evidence of 35 + *n*(3,4) 70	M1 A1 **[2]**	70 www gets B2.	

Question			Answer	Marks	Guidance	
5			The 10 routes shown from (0,0) to (3,2) each continue in one way via (4,2) to (4,3) and each continue in one way via (3,3) to (4,3) Hence 20	 B2 **[2]**	. www Need explanation SC B1 without explanation	Or, the 35 all pass through either 20(3,3) or 15 (4,2). 10 of the 20 at (3,3) come from (3,2) [and the rest from (2,3)] 10 of the 15 at (4,2) come from (3,2) [and the rest from (4,1)] So 10 + 10 = 20 are from (3,2) oe
6	(i)		$(-0.7, 5.3)$ $(-0.7, 0.7)$	B1 B1 **[2]**		SC B1 for (y =) 5.3 and 0.7.
6	(ii)			B1 **[1]**	A square with straight edges as shown with a vertex at (2,2). (and nothing more)	
7	(i)		B must lie on one of the two axes	B1 **[1]**	oe, say, horizontal and vertical lines from (0,0).	Do not accept a list of points unless an overall statement that includes **all** points is given (not just integers).

Question			Answer	Marks	Guidance	
7	(ii)		B lies on line $y = x$ or on line $y = -x$	B1 B1 **[2]**	oe	As **(i)**

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OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE **A2 7895-8**

Advanced Subsidiary GCE **AS 3895-8**

OCR Report to Centres

January 2013

4754 Applications of Advanced Mathematics

General Comments

This paper was of a similar standard to previous papers. The questions were accessible to all candidates but there were sufficient questions for the more able candidates to show their skills. Some candidates scored full marks and other high scores were seen. There were very few low scores as is the norm in the January series.

On Paper A questions 6, 8(ii) and 8(iii) scored least well.

The Comprehension was well understood and good marks were scored.

Candidates made similar errors as in previous papers. These included:

- Sign errors such as $-(x+1)= -x+1$ in Question 1
- Failure to include constants of integration in Question 8
- Poor anti-logging and rules of logarithms in Question 8
- Failure to read questions carefully as in Question 7(ii)
- Failure to give clear descriptions as in Comprehension Question 7
- Inappropriate accuracy, for example in Question 4b(i) giving say,7dp answers following working with 4dp values.
- Failure to give exact answers when required as in Questions3(i) and 4(a)
- Failure to give sufficient detail when verifying given results as in Question 8(ii).

Centres should be reminded that as Papers A and B are now marked separately it is essential that any additional sheets should be attached to the appropriate Paper to ensure that it is marked.

Comments on Individual Questions

Paper A

1 Most candidates understood the method for solving the equation involving algebraic fractions. The main errors were sign errors, especially $-(x+1) = -x+1$.

2 The method for finding the binomial expansion was understood by almost all candidates. Many candidates scored full marks here. The most common errors were sign errors, the omission of the validity or the use of $2x$ throughout instead of $(-2x)$.

3(i) This question was successfully answered by most candidates. Some failed to give their answers in exact form.

(ii) Most candidates used $y=\sin 2\theta = 2\sin\theta\cos\theta$ and many squared this but not all candidates subsequently used $\cos^2\theta = 1-\sin^2\theta$ to continue to the required result.

4(a) The method for finding the volume of revolution was usually correct. Many scored full marks in this part. There were a few errors in the integration (commonly either $\int 1\,\mathrm{d}x = 0$ or $\int e^{2x}\,\mathrm{d}x = 2e^{2x}$) but the main errors were failing to substitute the lower limit or giving the answer in inexact form.

(b)(i) Most candidates were able to use the trapezium rule, but many candidates gave their answers to an excessive degree of accuracy given that they were using values that were only given to 4dp.

(b)(ii) The explanations here were often excellent although a few were incomplete. Some said the trapezium rule was always an over-estimate whilst others failed to say that it was originally an over-estimate (or equivalent) in this case. Some used diagrams to illustrate their explanations with success.

5 Candidates seemed equally to choose the two approaches in the mark scheme to solve the trigonometric equation. Both were equally successful and few offered extra unnecessary solutions. The main error was to give insufficient accuracy in the final solutions.

Where solving tan θ=2 in degrees leads to θ=63.4º to 3sf,
giving θ= 1.11 radians = 63.598º (63.6º) and θ=1.1radians=63.0º were insufficiently accurate so we needed θ=1.107radians to achieve the same accuracy as 63.4º.

6(i) This question was answered well by the most able but many others could not cope with the fractions in part (i). AC was generally correct but often AD=cosφ/sinθ or sinθ/cosφ was the given answer, whilst others left their answer as a fraction within a fraction.

(ii) Good candidates were able to answer this with ease. Quite a few candidates made no response. Much depended upon their answers in part (i) which were followed through for the method marks. Those who then did not obtain the given answer should have realised that they ought to have reconsidered their answer to part (i).

7(i) The majority of candidates correctly found the length of the ridge of the tent.

Most candidates attempted to find the required angle using scalar products-often using an incorrect vector, particularly the vertical.

(ii) This part was well answered. Those who found two vectors in the plane and showed that they were both perpendicular to **i**–4**j**+5**k** and then proceeded to use x–4y+5z=d to find d and then a usually obtained full marks, although a few gave the equation as

i–4j+5z=16 and lost a mark.

Candidates should however be careful that they read the question carefully. They were asked to show that **i**–4**j**+5**k** was normal to the plane and **then**,… ' *find the equation of the plane'*. Some candidates started by trying to find the equation of the plane and did not attempt to show that the vector was normal. Other candidates decided to substitute the coordinates of points in the plane- in many cases without ever writing down the equation of the plane (for example, 1x0–4x–4+5x0=16) or mentioning a normal. These candidates are not showing that the vector is normal unless they provide a clear argument to support their calculations, nor are they finding the equation of the plane if they do not write it down in Cartesian form (or equivalent), 16 three times is not enough on its own.

Most candidates knew how to find a, either by substitution in the plane or by using scalar products, but some careless errors such as 4a=8, a=4 were seen. Some substituted +a instead of –a.

(iii) Those who substituted points were usually successful in verifying that the equation of the plane was $x+z=8$. Some derived the equation. Others showed that $\mathbf{i}+\mathbf{k}$ was perpendicular to two vectors in the plane but not all of these then went on to establish that the equation of the plane was $x+z=8$. Most scored the marks for finding the acute angle between the planes.

8(i) Most candidates correctly wrote down the value of h but quite a number failed to give the interpretation that the tree stopped growing when its height was 20m.

(ii) Those who approached the verification by integration were quite successful. The common errors were:-

- omitting the negative sign when integrating $1/(20-h)$
- omitting the constant of integration
- giving $\ln(h-20)$ in their answers (without modulus signs) despite having usually given $h=20$ as a maximum value in (i)
- incorrect anti-logging.

Those who approached from differentiation usually obtained some marks, particularly the mark for checking the initial conditions but many gave insufficient detail when verifying the given result.

(iii) There were a few completely correct solutions to this part. However, many different errors were seen from the majority of candidates. There was also a lot of confused work.

Those who started with the correct partial fractions, from $200/(20-h)(20+h)$ or $1/(20-h)(20+h)$, usually obtained the first three marks and then integrated having scored M1A1A1M1 thus far. Common errors then included omitting the negative sign when integrating $5/(20-h)$ (ie giving $5\ln(20-h)$ and hence A0) or failing to state and then evaluate a constant. Those who had no constant were unable to score further marks. Those who did score the first 5 or 6 marks (dependent upon when the constant was evaluated) often used the laws of logarithms correctly and anti-logged although some fiddled the signs when subsequently making h the subject.

Some candidates thought that $1/(400-h^2)=1/(h-20)(h+20)$. Marks were scored for using partial fractions on $1/(h-20)(h+20)$ but logarithms such as $\ln(h-20)$ for $h<20$ and constants such as $\ln(-1)$ could not obtain accuracy marks although the marks for anti-logging and making h the subject were still available.

There were also a number who felt that $1/(400-h^2)=1/(200-h)(200+h)$.

The use of modulus signs was rarely seen.

(iv) Usually correct.

(v) Most candidates scored all three marks.

Paper B

1 Most candidates scored all three marks.

2 This was generally well answered although some just gave one point. Some got the sloping part right but had vertical, not horizontal, lines. Others had the right horizontals but the sloping line was flatter.

3(i) This was usually correct although some stated t(P,A)=t(P,B)=3 which was wrong.

(ii) t(P,A)=3 was usually given.

4 Many obtained both marks although some left their answer as $n(3,4)+n(4,3)$ without evaluation.

5 This was less successful. Many gave the answer 10 as $n(3,2)$ is 10. Others gave 10+2=12.

6 There were some wrong answers with unclear methods in part (i) although there were also many correct solutions. Part (ii) was usually right.

7 There were many correct solutions using mathematical terms such as the x-axis, y-axis, $x=0$,$y=0$,$y=x$ and $y=-x$. Some others gave appropriate descriptions such as all points $(p,0)$ for all real values p or said all points vertically above and below and horizontally from A or equivalent whilst others used the points of the compass in their descriptions. Some only gave a list of points which was insufficient without a general statement to include all points. Some only gave integer values. In several cases one felt that the candidates probably did know the correct answer but were unable to explain it clearly.